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THE COSMOLOGICAL CONSTANT, BROKEN GAUGE THEORIES AND 3°K BLACK-BODY RADIATION

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Abstract

It is suggested that the cosmological constant today be identified with the one provided by gauge theories at $T = 3°K$ and not $T = 0°K$ or $T = 10^{15}°K$ as done so far. We then calculate $\Lambda(3°K)$ in the σ-model, W-S model and Mohapatra and Sidhu model. It is found that an excellent value is obtained within the σ-model; a vanishingly small Λ in the W-S model. For the M-S model, Λ can be fitted only if at least one of the Higgs meson has a very light mass ($\approx 3°K$).

The introduction of the cosmological constant Λ about sixty years ago by Einstein lost its initial motivation after the discovery of the expansion of the universe. Today, however, a non zero Λ is neither excluded by observations (which only demand $\Lambda \leq 10^{-57}\ \mathrm{cm}^{-2}$) nor contradicted by any existing physical principle. Attempts to solve some cosmological problems in the late 1960s revived the interest in the subject[1]. Recently, a reexamination[2] of the deceleration parameter, taking into account evolutionary corrections, favored a negative value of q_0, thus implying an accelerating universe, i.e. $\Lambda > 0$.

From the theoretical side, physicists are unhappy about such an ad hoc quantity unless a derivation from a microscopic theory can be provided. Zel'dovich[3] first pointed out that Λ can be interpreted as $8\pi G$ times the zero-point energy momentum tensor $T_{\mu\nu}^{(v)}$ of the vacuum, which is Lorentz invariant. After the successful invention of spontaneously symmetry-breaking gauge theories, it was realized that the vacuum is really a medium, a condensate of (scalar) particles. The asymmetry in physical laws is ascribed to the asymmetrical nature of the vacuum. The order parameter is characterized by the vacuum expectation value σ of the Higgs field $\varphi = \sigma + \varphi'$. Similarly, the energy-momentum tensor is decomposed into a c-number part $T_{\mu\nu}^{(c)}$ and an operator part $T_{\mu\nu}^{(M)}$. The latter corresponds to the matter part of $T_{\mu\nu}$ in the Einstein equation, while the former gives rise to the Λ-term. In the $V_0(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$ theories, we have

$$
\begin{aligned}
T_{00}^{(c)} &= \varepsilon(\sigma) = \frac{1}{4}\lambda\left(\sigma^2 - \sigma_0^2\right)^2 + \varepsilon_0 \\
\Lambda &= \frac{8\pi G}{c^4}\, T_{00}^{(c)}(\bar{\sigma})
\end{aligned}
\tag{1}
$$

where $\sigma_0^2 = \mu^2/\lambda$ and ϵ_0 is an arbitrary additive constant. The equilibrium $\bar{\sigma}$ of the Higgs field has to be determined from the condition $\partial V/\partial\sigma = 0$, where the effective potential $V(\sigma)$ is given by[4] $V(\sigma) = V_0(\sigma) + \hbar V_1(\sigma)$: V_0 is the classical tree approximation ($\hbar \to 0$), whereas $V_1(\sigma)$ represents radiative quantum corrections.

When $-\mu^2 < 0$, the origin $\sigma = 0$ (symmetric vacuum) becomes a local maximum of $V(\sigma)$, while the true minimum (asymmetric vacuum) shifts to a non-zero $\bar{\sigma}$ which corresponds to the real vacuum.

Within the classical tree approximation (in the sense $\hbar \to 0$), $V(\sigma) = V_0(\sigma)$, $\bar{\sigma} = \sigma_0$ and so the cosmological constant for the equilibrium vacuum becomes

$$\Lambda(\bar{\sigma}) = \Lambda(\sigma_0) = \frac{8\pi G}{c^4}\,\varepsilon_0 \qquad (2)$$

A small Λ ($\leq 10^{-57}$ cm^{-2}) implies a small ϵ_0. The arbitrariness in ϵ_0 is intrinsic to the field theory, where only differences in energies are physically significant.

Kirzhnits and Linde[5] simply stated that ϵ_0 must be less than 10^{-28} gm/cm^3 for Λ to be of the order of 10^{-57} cm^{-2}.

A second possibility was investigated by Dreitlein[6], who postulated that the energy of the symmetric vacuum be zero: $\epsilon(\sigma = 0) = 0$, and so

$$\Lambda(\bar{\sigma}) = \Lambda(\sigma_0) = -8\pi G\lambda\sigma_0^4/4c^4 = -8\pi G m_\varphi^2/8\sqrt{2}\,c^4 G_F \qquad (3)$$

where in the last step use was made of the definition of λ and σ as from the Weinberg-Salam model[7]. There are several unfortunate consequences. For one thing, Λ is negative. Moreover, using values for m_φ as from Ref. (8,9), it follows that Λ is $\sim 10^{-6}$ cm^{-2}, about 50 orders of magnitude larger than the limit observed today.

Since the real vacuum is asymmetric in all kinds of theories with broken symmetries, it seems quite natural to define the energy density of the asymmetric vacuum to be zero, i.e. $\epsilon(\sigma = \bar{\sigma}) = 0$. Alternatively, the cosmological constant for the asymmetric vacuum is assumed to be zero.

Then, how could we explain the positive Λ favored by observations during recent years? For this, we note that in the above discussion, the cosmological constant was calculated in the zero-temperature field theory, while the actual ambient temperature in the universe today is approximately 3°K. To incorporate temperature effects, we invoke the temperature dependent field theory. In analogy with superconductivity, Kirzhnits[5] argued that $\bar{\sigma}$ is a function of temperature, $\bar{\sigma} = \sigma(T)$, which vanishes at $T > T_c$. Thus, in the "hot universe" model, Λ is actually not a constant, but a temperature dependent parameter $\Lambda(T)$. The huge value $\sim 10^{-6}$ cm^{-2}, can now be thought of as being applicable at $T > T_c \sim 10^{15}$ °K, which occurred in the early phases of the universe. Since the universe today is cold, we propose in this paper that the cosmological constant observed today be identified with

$$\Lambda_{today} = \Lambda(3°k) = \frac{8\pi G}{c^4} \frac{\lambda}{4} \left[\sigma^2(T) - \sigma_o^2 \right]^2_{T=3°k} \qquad (4)$$

The tiny Λ_{today} is then the consequence of the nearly zero temperature of the present day universe. We should not identify Λ_{today} either with $\Lambda(0°K)$ which is zero or with $\Lambda(T_c)$ which is extremely large.

In what follows we shall calculate $\Lambda(3°K)$ in various models.

1. σ-Model. In the simplified version of the σ-model, we have a complex scalar field $\sqrt{2}\varphi = \varphi_1 + i\,\varphi_2$, corresponding to the Lagrangian density

$$\mathcal{L} = \mu^2 \varphi\varphi^* + \partial_\mu\varphi\, \partial^\mu \varphi^* - \lambda(\varphi\varphi^*)^2 .$$

After symmetry breaking, one of the scalar fields develops a non-vanishing vacuum expectation value, say $\langle\varphi_1\rangle = \sigma \neq 0$. Within the one loop approximation, we have the following self consistently coupled equations[5] for σ, m_1 and m_2

$$\sigma(\mu^2 - \lambda\sigma^2 - \lambda\Pi) = 0 \quad ; \quad m_1^2 = -\mu^2 + \lambda(3\sigma^2 + \Pi)$$
$$m_2^2 = -\mu^2 + \lambda(3\sigma^2 + \Pi) \tag{5}$$

where Π is the temperature-dependent part of the self energy graph, given by $(E^2 = k^2 + m^2)$

$$\Pi = 3F_+(T, m_1) + F_+(T, m_2) \quad ; \quad F_\pm = \int \frac{d^3k}{(2\pi)^3} \frac{(2E)^{-1}}{\exp \beta E \pm 1} \tag{6}$$

For the ordered state, $\sigma \neq 0$, $m_1^2 = 2\lambda\sigma^2$, $m_2 = 0$, and $\sigma(T)$ is determined by the nonlinear equation $(F = T^2 f)$

$$\sigma^2 = \sigma_0^2 - T^2 \left\{ f_+(0) + 3 f_+ \left(\frac{\sqrt{2\lambda}\,\sigma}{T} \right) \right\} \tag{7}$$

We have solved (7) numerically and the results are shown in Fig. 1.

In the low temperature regime ($T^2 << \lambda\sigma_0^2$), this equation can be solved iteratively, yielding, [f(0) = 1/12]

$$\Lambda(T) = 3.3\ 10^{-63}\ \lambda \left(\frac{T}{3°K}\right)^4 \left[1 + O\left(\exp -m_1/kT\right)\right]\ \mathrm{cm}^{-2} \tag{8}$$

In this formula, we note that the contribution of φ_2 (Goldstone mode) dominates over that of φ_1 (massive Higgs field). The condition $\Lambda_{today} \lesssim 10^{-57}$ cm^{-2} implies that $\lambda < 3\ 10^5$.

2. Weinberg-Salam Model[7]. Within the one loop approximations, we have derived the following equations for the determination of $\sigma(T)$

$$\begin{aligned} \sigma^2 = \sigma_0^2 - 3F_+(T, m_\varphi) - \left(1 + 3\frac{m_Z^2}{\lambda\sigma^2}\right) F_+(T, m_Z) - 2\left(1 + 3\frac{m_W^2}{\lambda\sigma^2}\right) F_+(T, m_W) \\ + 4\frac{m_e^2}{\lambda\sigma^2} F_-(T, m_e) + 4\frac{m_\mu^2}{\lambda\sigma^2} F_-(T, m_\mu) \end{aligned} \tag{9}$$

where

$$m_W = \frac{1}{2} g\sigma, \quad m_Z = \frac{1}{2}\sigma(g^2 + g'^2)^{1/2}, \quad m_\varphi = \sigma(2\lambda)^{1/2}, \quad m_e = 2^{-1/2}\sigma G_e$$

In the low temperature regime, the electron contribution dominates because of its small mass, and so we can write

$$\Lambda(T) = 4.8\; 10^{-61}\, \lambda \left(\frac{T}{3^\circ K}\right)^4 \left(\frac{4 m_e^2}{\lambda \sigma^2}\right)^2 e^{-2m_e/kT} \left(\frac{1}{32\pi^3}\, \frac{m_e}{kT}\right)$$

$$= \Lambda(\sigma\text{-model}) \left(\frac{48\, m_e^2}{\lambda \sigma^2}\right)^2 \frac{1}{32\pi^3}\, \frac{m_e}{kT}\, e^{-2m_e/kT} \qquad (10)$$

At $3^\circ K$, the exponential is very small ($m_e/kT \sim 10^{10}$) and so $\Lambda \approx 0$.

Therefore, the cosmological constant within the Weinberg-Salam model is vanishingly small compared to the one obtained in the σ-model.

3. The Mohapatra and Sidhu Model[(10)]. In Figure 2, we show the tadpole diagrams contributing to Λ. In the σ-model, $m\phi_2 = 0$. In the W-S model, the mass of the lightest particle coupled to the Higgs field is $m_e \sim 0.5\, M_{ev}$ and this reduces Λ drastically. The M-S model[(10)] allows the graph 2c, where neutrinos have a tiny arbitrary mass through the Yukawa coupling to the Higgs field. This will increase the very small exponential of (10) to about unity. The cosmological constant can be computed to be

$$\Lambda(T) = 4.8\; 10^{-61} \left(\frac{T}{3^\circ K}\right)^4 \left\{ \frac{h\, m_\nu}{\langle m_\varphi \rangle}\, f_-\!\left(\frac{m_\nu}{kT}\right)\right\}^2 \qquad (11)$$

where h is the strength of the Yukawa coupling and $\langle m_\varphi \rangle$ represents some typical mass for the Higgs field.

The vanishing small mass of the neutrinos is seen to be only a necessary condition for a $\Lambda \sim \Lambda_{today}$. In fact, we must also have an appropriate coupling constant. Taking $m_v \sim kT \sim 10^{-4}$ ev, in order for f_- to be of the order one, we must also require that the factor

$$h \, 10^{-10} / \langle m_\varphi (\text{Mev}) \rangle$$

not be exceedingly small. As of today, the M-S model has not been yet thoroughly investigated to allow us to fix the values of h and $\langle m_\varphi \rangle$ in any reliable way. In fact, all the determinations of m_φ (11) have been based on the W-S model, where there is only one Higgs boson. In the M-S model, there are ten Higgs scalars and it is not a priori clear whether or not the values for m_φ quoted in Fig. 3 of Ref. 11 apply to all of them. This leaves the possibilities that at least one of the 10 Higgs boson could have a small mass, thus making $\Lambda \sim \Lambda_{today}$. We can reverse the argument and conclude that for the M-S model to reproduce a reasonably large Λ, one of the Higgs basons must have a very small mass.

In conclusion, we would like to add the following remarks. Short of accepting ad hoc hypotheses like the one of Ref. 5, no satisfactory derivation has so far been proposed for the cosmological constant, except the one presented here.

The σ-model does yield acceptible results, the W-S model too small a value for Λ and the M-S model holds good promises. If we also consider that the M-S model has partially been motivated by unsatisfactory predictions of W-S model (especially for atomic physics), the present analysis indicates one more difficulty of cosmological nature, which can in principle be cured in the M-S model.

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Figure Captions

Fig. 1. The $\sigma(T)$ vs. T function. At high temperatures the symmetry is restored, i.e. $\sigma \to 0$.

Fig. 2. The dominant tadpole diagrams: (a) σ-model, (b) W-S model and (c) M-S model.

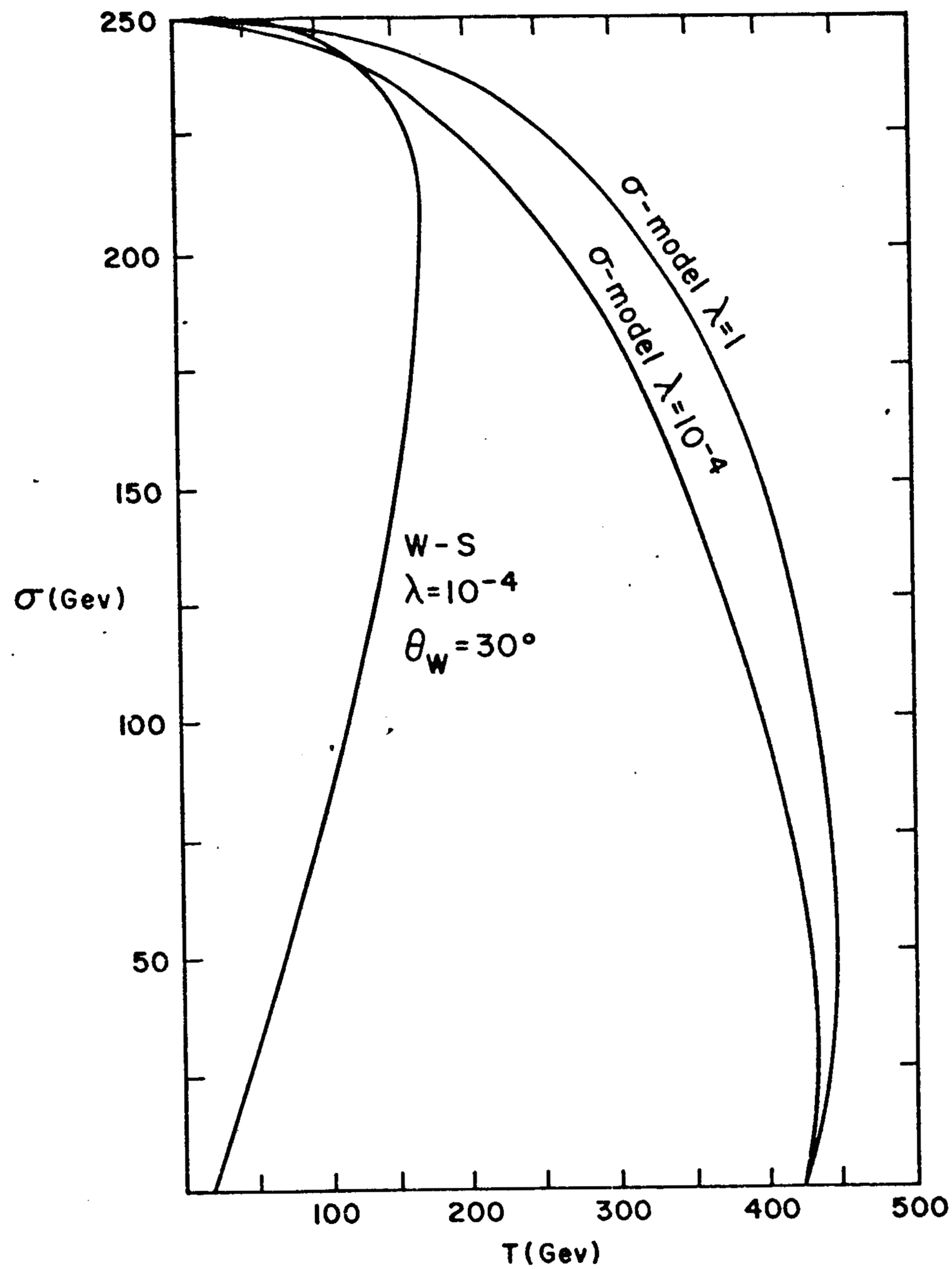
250
200
150
100
50
σ (Gev)
100
200
300
400
500
T (Gev)
W - S
λ=10^-4
θ_W=30°
σ-model λ=1
σ-model λ=10^-4

Fig.1

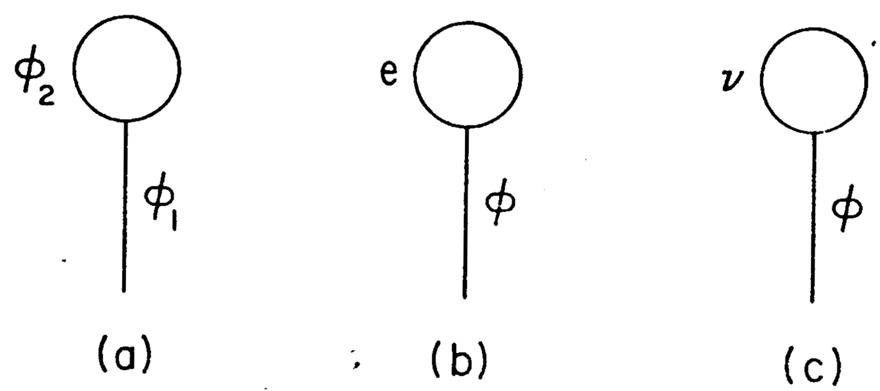

Fig. 2